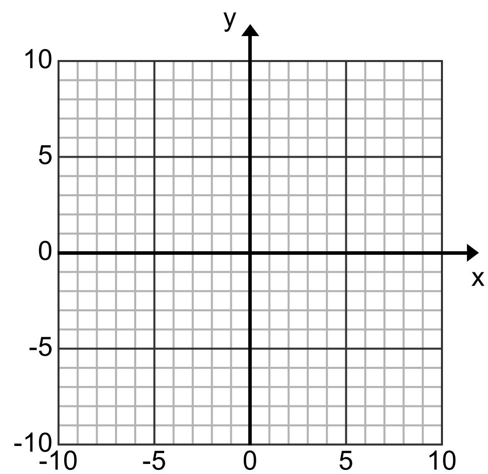
Unit 5: Trig Inverses and Solving Trig Equations

Recall that the <u>inverse</u> of a function is found by interchanging the x- and y- coordinates. A function and its inverse are then reflections of each other across the line y = x.

Example: Give 5 ordered pairs on the graph of $y=(x-3)^3+1$ Then give 5 points on the graph of it inverse. Graph the function and its inverse. Is the inverse a function?

x	y

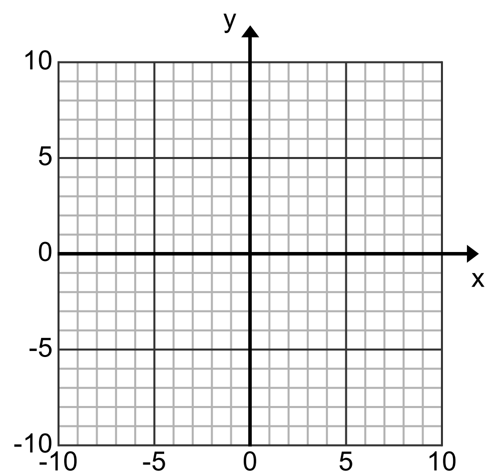
y	x



Example: Give 5 ordered pairs on the graph of $y=|x+2|$ Then give 5 points on the graph of it inverse. Graph the function and its inverse. Is the inverse a function?

x	y

y	x

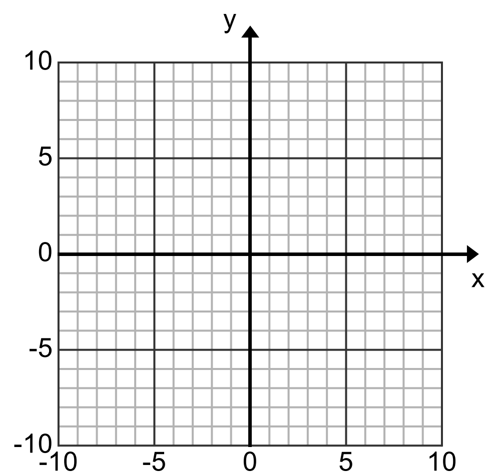


In order to make sure that the inverse of a function is also a function, we can specify a restricted domain for the original function. Suggest a restricted domain for $y = |x + 2|$

Example: Give 5 ordered pairs on the graph of $y = x^2 - 4$ with the domain restricted to $x \geq 0$. Then give 5 points on the graph of its inverse. Graph the function and its inverse. Is the inverse a function?

x	y

y	x



Solving Trig Equations on the Unit Circle

We know that the cosine and sine functions map each real number, t, onto a point (x,y) on the unit circle.

Use your knowledge of the unit circle to evaluate each trigonometric expression.

$$\sin\frac{\pi}{3} \qquad \tan\pi$$

$$\cos\frac{\pi}{4} \qquad \sin\frac{-\pi}{6}$$

The inverse trig relations map each point on the unit circle back to one or more real numbers, t. We use the notation $t=\arccos(x)$ or $t=\cos^{-1}(x)$ to mean that we want the real number t whose cosine is x.

Example: Rewrite each equation using inverse notation. Then find the exact value(s) of t which make the equation true on the interval $0\le t<2\pi$.

$$\sin t=1 \qquad \cos t=\frac{-1}{2}$$

$$\tan t=1 \qquad \sin t=\frac{\sqrt{2}}{2}$$

Since the sine and cosine functions are periodic, there are actually an infinite number of t values which satisfy each equation.

Example: Find all real values of t which make each equation true.

$$\sin t = -1 \qquad \cos t = \frac{-1}{2}$$

$$\tan t = \sqrt{3} \qquad \sin t = \frac{-\sqrt{3}}{2}$$

Of course, our knowledge of the unit circle will only allow us to solve for certain values of t. For other solutions, we will need to use a calculator.

Example: Use your calculator to help find all real values of t.

$$\sin t = \frac{1}{4} \qquad \cos t = \frac{-2}{3}$$

$$\tan t = 2 \qquad \sin t = -2$$

Solving Trig Equations – Principal Solutions

As we have seen, the inverse of the trig functions are not themselves functions. However, we can make the inverse trig relations into functions by restricting the domain of the original functions. The restricted domains are called Principal Values.

Function	Domain	Range
y = sin(x)	$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$	$-1 \le y \le 1$
y = arcsin(x) or y = $\sin^{-1}(x)$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
y = cos(x)	$0 \le x \le \pi$	$-1 \le y \le 1$
y = arccos(x) y = $\cos^{-1}(x)$	$-1 \le x \le 1$	$0 \le y \le \pi$
y = tan(x)	$-\frac{\pi}{2} < x < \frac{\pi}{2}$	All real numbers
y = arctan(x) y = $\tan^{-1}(x)$	All real numbers	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

Example: Give the Principal solutions

a) $\arcsin\left(\frac{-1}{2}\right)$

b) $\cos^{-1}\left(-\frac{1}{2}\right)$

What value does your calculator give for $\arcsin\left(\frac{-1}{2}\right)$ and $\cos^{-1}\left(-\frac{1}{2}\right)$?

The key to solving more complicated trig equations, as for any algebraic equation, is to <u>isolate the variable</u>. Isolating the variable frequently requires factoring and the Zero Product Property. In addition, to solve trig equations, you will use some or all of the techniques we learned last chapter, including the Pythagorean identities and the sum/difference/double angle formulas.

Like last chapter, you may not be sure at first what technique to try. TRY SOMETHING!!!

Example: Solve $2\sin^2 x + \sin x - 1 = 0$ for principal values of x.

Example: Solve $\sin 2x - \cos x = 0$ for principal values of x.

Solving Trig Equations – General Solutions

You already know that a trig identity is true for all values of x. A trigonometric equation (like an algebraic equation) is only true for some values of x. Because trig functions are periodic, most trig equations have more than one solution. Sometimes, you will be told to restrict the solutions to the Principal Values, or some other restricted Domain.

Equation	Solution	Explanation
$\sin^2 x + \cos^2 x = 1$	All real x	This is an identity
sin x = ½	x = $\frac{\pi}{6} + 2\pi k$ x = $\frac{5\pi}{6} + 2\pi k$	There are an infinite number of solutions
$\sin x = \frac{1}{2}$, for principal values of x	x = $\frac{\pi}{6}$	Only solution where $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$
$\sin x = \frac{1}{2}$, for $0 \le x < 2\pi$	x = $\frac{\pi}{6}, \frac{5\pi}{6}$	Two solutions in the given interval

Be sure to read each question carefully so you know what type of answer you are being asked to find.

Example: Solve $\sin^2 x - \sin x + 1 = \cos^2 x$ for $0 \le x < 2\pi$

Example: Solve $2\cos^4 x + 3\sin^2 x = 2$ for all real values of x. (This is called the general solution).

Example: The height h (in feet) above ground of a seat on a Ferris wheel at time t (in minutes) can be modeled by $h(t) = 53 + 50\sin\left(\frac{\pi}{16}(t-8)\right)$.

a) How long does it take to go around the ride one time?

b) If the ride lasts 96 seconds, at what times will a rider be at the maximum height?

Solving Trig Equations/Inequalities – Graphing Calculator

Some trig equations are difficult or impossible to solve algebraically. A graphing calculator can be helpful.

Example: Solve $6\tan^2 x - 5\tan x - 6 = 0$ by factoring. Then use your calculator to find all real solutions.

Example: Solve by graphing: sin x = 2 cos x for $0 \le x < 2\pi$

Graphs can also help you visualize solutions to trig inequalities. Start by solving the related equality. Then examine the graph to find the solutions to the inequality.

Example: Solve $2\sin\theta + 1 < 0$ for $0 \le \theta < 2\pi$

Example: The monthly sales S (in hundreds of units) of skiing equipment at a sports store are approximated by $S = 58.3 + 32.5\cos\left(\frac{\pi}{6}t\right)$ where t is the time in months. During what months will the sales exceed 7000 units?