Analysis CP Unit 1: Circular and Right Triangle Trig

Lesson 4-1 Angles and Degree Measures

An angle is formed by two rays joined at the vertex. An angle with its vertex at the origin and its initial ray along the positive x-axis is said to be in standard position. A positive angle is formed when the rotation of the terminal ray is in the counter-clockwise direction. A negative angle is formed when the rotation is in the clockwise direction.

Example: Sketch an angle in standard position that measures -135°.
In what quadrant is its terminal ray?

An angle of measure 360° represents a complete rotation around a circle. Two angles in standard position are called coterminal when they have the same terminal ray.

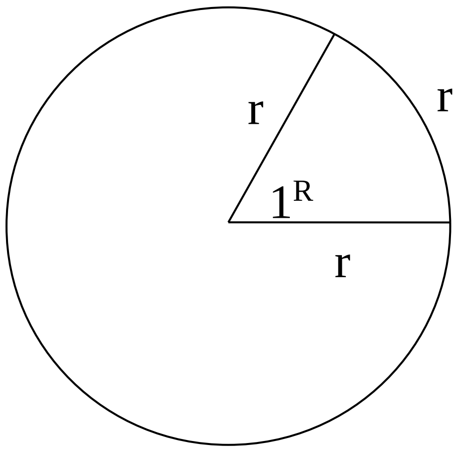
Example: Name a positive angle that is coterminal with -135°

Example: Name a negative angle that is coterminal with -135°

Clearly, there are an infinite number of angles that are coterminal with -135°. These angles can be represented as $-135+360k$, where k is an integer.

Your Turn: An angle of -775° is in standard position. Determine a coterminal angle between 0 and 360° and state the quadrant of the terminal side.

One unit for measuring angles is the degree. A complete rotation around a circle measures 360°. Another unit for measuring angles is the radian.



One radian is defined as the measure of the central angle formed by a circular arc that is the length of the radius.

Do you think 1 radian is a little less or a little more than 60°? Explain.

For a circle with radius 1, when the central angle is 360°, then the measure of the arc is the circumference of the circle, or 2π. This gives the conversion factor:

$$2\pi \text{ (radian)} = 360^\circ \quad or \quad \pi \text{ (radian)} = 180^\circ$$

Example: Convert 45° to radian

$$45^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{\pi}{4}$$

Your turn: Convert 270° to radian

Convert $\frac{7\pi}{6}$ to degrees

Find the exact degree measure of 1 radian.

The degree can be subdivided into 60 equal parts known as minutes. Minutes can be further subdivided into 60 equal parts called seconds. We can convert degree measures in Degrees, Minutes, Seconds (DMS) format to decimal format and vice-versa.

Example: Convert 12° 31’ 12’’ to decimal degrees

$$12^\circ + 31'\left(\frac{1^\circ}{60'}\right) + 12''\left(\frac{1^\circ}{3600''}\right) = 12.520^\circ$$

Example: Convert 12.520° to DMS

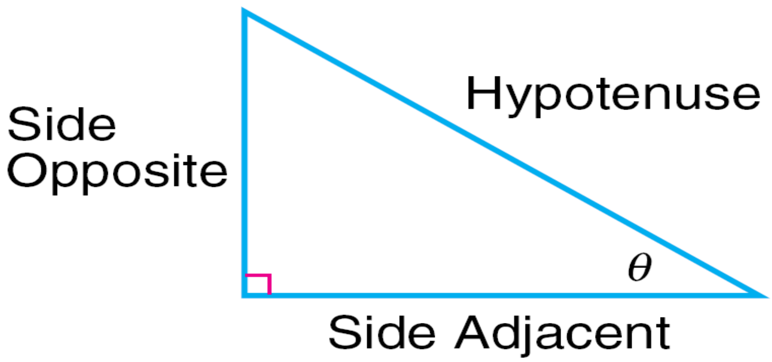
$$12^\circ + 0.520^\circ\left(\frac{60'}{1^\circ}\right) = 12^\circ 31.2'$$

$$12^\circ + 31' + 0.2'\left(\frac{60''}{1'}\right) = 12^\circ 31' 12''$$

Your Turn: Convert -29.44° to DMS

Lesson 4.3: Trigonometric Ratios in Right Triangles

You are already familiar with the three most important trigonometric ratios: sine, cosine and tangent.



$$\sin\theta = \frac{opposite}{hypotenuse} \qquad \cos\theta = \frac{adjacent}{hypotenuse} \qquad \tan\theta = \frac{opposite}{adjacent}$$

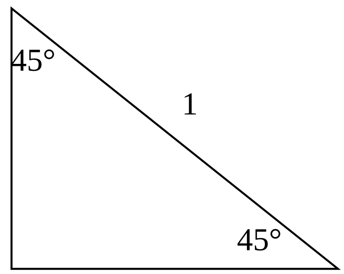
There are three more trigonometric ratios you will need to know: cosecant, secant, and cotangent.

$$\csc\theta = \frac{hypotenuse}{opposite} \qquad \sec\theta = \frac{hypotenuse}{adjacent} \qquad \cot\theta = \frac{adjacent}{opposite}$$

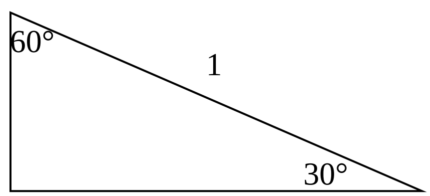
These three ratios are also called the <u>reciprocal identities</u> because

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Example: Use the 45-45-90 special right triangle to find <u>exact values</u> for the six trigonometric ratios of a 45° angle. Assume that the length of the hypotenuse is 1.



Your turn: Use the 30-60-90 triangle to find <u>exact values</u> of the six trigonometric ratios for a 30° and 60° angle.



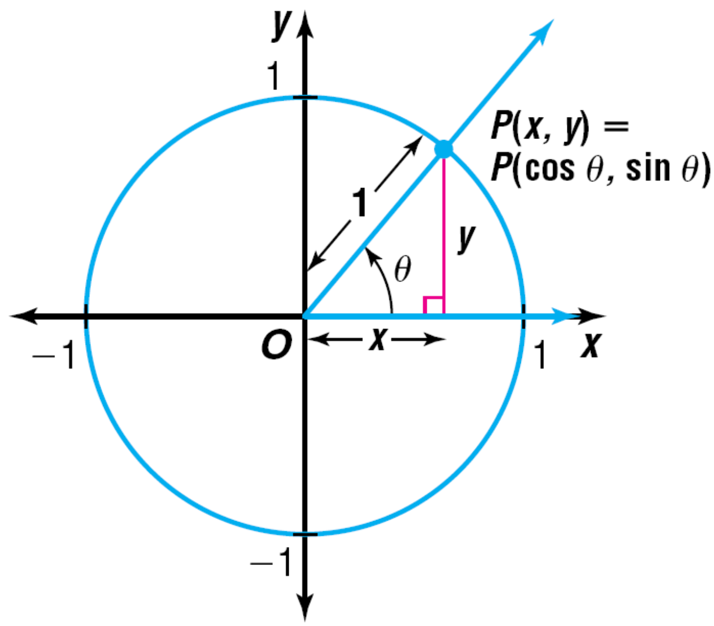
Summarize your results in the table below:

θ	Sin θ	Cos θ	Tan θ	Csc θ	Sec θ	Cot θ
30°						
45°						
60°						

Lesson 4.4: Trig Functions on the Unit Circle

The ***unit circle*** is a circle of radius 1, centered at the origin. Suppose that some angle θ is formed when the terminal side of an angle passes through some point $P(x, y)$ on the unit circle. Since the hypotenuse of the formed triangle is 1, then the following trig functions are true:

sin θ = y   and cos θ = x



It follows that

tan θ =

csc θ =

sec θ =

cot θ =

Example: Use the unit circle to find the exact values of the six trigonometric functions for a 60° angle.

Example: Evaluate $\sin^2\theta + \cos^2\theta$ for $\theta = 30^\circ, 45^\circ, 90^\circ$

Make a conjecture about the value of $\sin^2\theta + \cos^2\theta$ for any θ

A <u>reference angle</u> is the acute angle formed by the terminal side of an angle and the x-axis. The measure of a reference angle is *<u>always acute and positive</u>*.

Example: Draw each angle in standard position and find the measure of its reference angle.

a) 45°

b) 135°

c) 225°

d) 315°

Summary: For angle α in standard position with its terminal side in the given quadrant, the reference angle is

Quadrant	I	II	III	IV
Reference Angle ° Radian	α	$180-\alpha$ $\pi-\alpha$	$\alpha-180$ $\alpha-\pi$	$360-\alpha$ $2\pi-\alpha$

When finding the reference angle, you should first find the coterminal angle between 0 and 360° or 0 and 2π radian.

Your turn: Find the reference angle for each given angle.

a) 575°

b) $-\frac{7\pi}{6}$

You can use reference angles and the unit circle to find trig ratios of any angle.

Example: Find the 6 trig ratios for $\frac{5\pi}{3}$

Your turn: Find the 6 trig ratios for 405°

Example: Evaluate $\dfrac{4\sin\left(\frac{\pi}{2}\right)-\cos^2\left(-\frac{\pi}{2}\right)}{4\tan\left(\frac{\pi}{4}\right)}$

Your turn: Evaluate $\dfrac{4\sin^2\left(\frac{\pi}{6}\right)-\cos(-\pi)}{\tan\left(\frac{\pi}{3}\right)}$

Lesson 4.4 continued: Trig Functions for any Angle

We can extend the concept of the Unit Circle to triangles that do not have a hypotenuse of 1 unit.

Example: Find the values of the six trigonometric functions for an angle θ with terminal side passing through (-5, 12)

Example: If sin $\theta = -\frac{4}{5}$ and the terminal side lies in Quadrant III, find the remaining five trigonometric values for θ.

Example: The terminal side of angle α lies in quadrant II and is coterminal with the line $y = -\frac{1}{3}x$. Find the values of sin α, cos α, and tan α.

Example: You are riding the Ferris wheel at the local carnival. You board the ride at the bottom of the wheel, which is 4 feet above the ground. The radius of the Ferris wheel is 36 feet.

a) How far above the ground will you be when you reach the top of the wheel?

b) After you board at the bottom, the wheel rotates 120° counterclockwise to let another passenger on. How far above the ground are you?

c) From your position at 120° the wheel rotates another 180° counterclockwise. How far above the ground are you now?

Lesson 4.7: Using Inverse Trig Operations

Use your Unit Circle plate to answer the questions below:

If $\sin\theta = \frac{1}{2}$, what could be the value(s) of θ?

If $\tan\theta = 1$, what could be the value(s) of θ?

To find the measure of an angle when you know the value of one of its trig functions, you use the <u>inverse</u> operation. We write the inverse of sine as $\sin^{-1}$ or arcsin. We will learn later that the input value of the angle must be restricted in order for the inverse function to have one unique solution.

Example: Solve each equation for $0 \le \theta < 360^{\circ}$

$$\cos\theta = \frac{-1}{2}$$

$$\tan\theta = 1$$

$\sin\theta = .75$ (using calculator)

Example: Solve each equation for $0 \le t < 2\pi$

$\sin t = -1$

$\tan t = \frac{\sqrt{3}}{3}$

$\cos t = -\frac{1}{4}$ (using calculator)

$\sin t = 2$ (using calculator)

Example: Solve $2\tan^2\theta - 5 = 1$. Assume θ is in quadrant I.

Example: Evaluate each expression. Assume that all angles are in quadrant I.

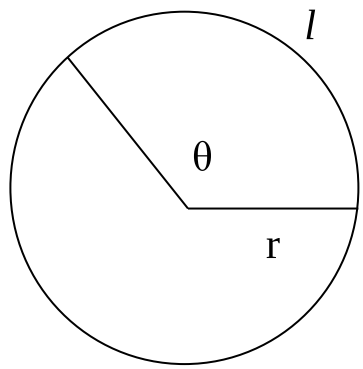
$\tan\left(\cos^{-1}\frac{4}{5}\right)$

$\tan\left(\arctan\frac{6}{11}\right)$

Lesson 4.1, continued: Arc Lengths and Area

We know from Geometry that the circumference and area of a circle are given by $C = 2\pi r$ and $A = \pi r^2$

A <u>sector</u> of a circle is a region bounded by a central angle θ and its intercepted arc, *l*. The length of the arc and the area of the sector are proportional to the corresponding circumference and area of the circle.



Arc Length = $k(2\pi r)$

Area of Sector = $k\pi r^2$

For angles measured in degrees, $k = \frac{\theta}{360}$

For angles measured in radian, $k = \frac{\theta}{2\pi}$

Example: Find simplified expressions for the arc length and sector area when θ is measured in radian.

Example: A sector of a circle has central angle $\frac{\pi}{3}$ and radius 5 cm. Find its arc length and area.

Example: Find the distance traveled by the minute hand of a watch as it goes from 1:00 to 1:25. Assume that the radius of the minute hand is ¾ cm.

Example: A sector of a circle with radius 5 cm has an area of $10\pi\ cm^2$. Find the measure of its central angle in both degrees and radian.

Example: Your mother just baked a delicious 14-inch diameter apple pie. She will cut you a slice with a central angle of 40°. What is the area of your sector of the pie?