Lesson 7-2: Inverse Functions and Relations

Objectives: To find the inverse of functions or relations
To determine the graphical relationship between a function and its inverse

Warm Up: State the domain and range of

a) (1, 2) (3, 4) (5, 6)

b) $f(x) = x + 6$

c) $y = x^2 + 5$

Recall that a relation is a set of ordered pairs. The <u>inverse relation</u> is the set of ordered pairs obtained by reversing the coordinates of each ordered pair.

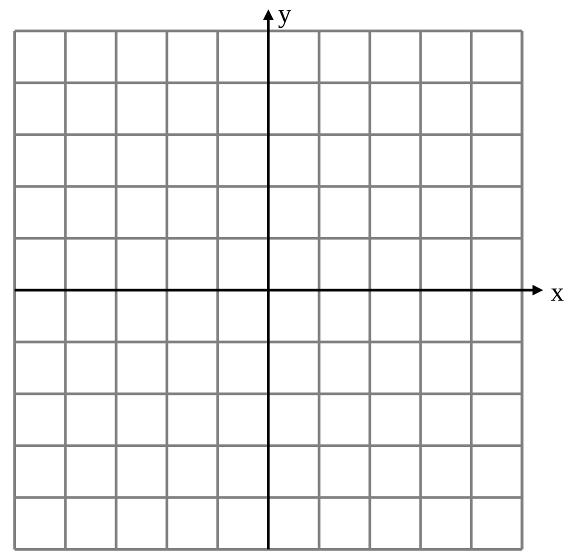
Example: Find the inverse of the relation (1, 2) (3, 4) (5, 6). What is the domain and range of the inverse?

To find the inverse of a function, $f(x)$:

Step 1: Write the original equation with y instead of $f(x)$
Step 2: Interchange the x and y.
Step 3: Solve for y.
Step 4: If the inverse relation is a function, replace y with $f^{-1}(x)$

Example: Find the inverse of $f(x)=3x+5$. Is the inverse a function?

Graph both $f(x)$ and $f^{-1}(x)$ on the same coordinate plane.



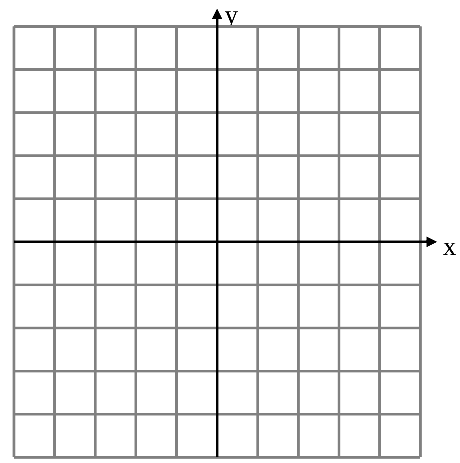
Name three points that are on $f(x)=3x+5$

Show that the inverse of each point is on $f^{-1}(x)$

Note that a function and its inverse are reflections of each other over the line y = x.

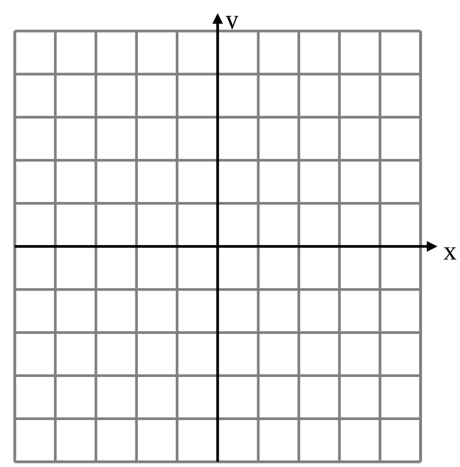
The inverse of a function is not always another function.

Example: Find the inverse of $f(x) = x^2$. Graph $f(x)$ and its inverse. Is the inverse a function? Why or why not?



If we restrict the range (output) of the inverse relation to nonnegative numbers, then we have the square root function, $f(x) = \sqrt{x}$.

Example: Find the inverse of $f(x) = x^2 + 2$. Graph $f(x)$ and its inverse. What are the domain and range of each relation?



Lesson 7-2, continued: Composition and Inverse Functions

Objectives: To determine whether two functions or relations are inverses using composition of functions

Composition of Functions

If f(x) and g(x) are two functions then their composite function is defined as

$$[f \circ g](x) = f[g(x)]$$

Example: If $f(x) = x^2 - 1$ and $g(x) = x + 3$, find

$[f \circ g](x)$

$[g \circ f](x)$

$(f \circ g)(4)$

$(g \circ f)(4)$

In general, $[f \circ g](x) \neq [g \circ f](x)$

Example: If $g(x) = \sqrt{x}$ and $h(x) = x^2 + 6x + 9$ find

$(g \circ h)(x)$

$(h \circ g)(x)$

However, if f(x) and g(x) are inverses, then

$$f[g(x)] = g[f(x)] = x$$

Example: Use composition of functions to determine if $f(x) = 5x + 10$ and $g(x) = \frac{1}{5}x - 2$ are inverses.

Example: Use composition of functions to determine if $g(x) = 4x - 3$ and $h(x) = 4x + 3$ are inverses.

Composition of Functions – Extra Example

In the mail you receive a coupon for $5 off a pair of jeans. When you arrive at the store, you find that all jeans are 25% off.

a) You find a pair of jeans you like for x dollars. Write a function f(x) that represents the cost of the jeans after the $5 off coupon.

b) Write a function g(x) that represents the cost of the jeans after the 25% discount.

c) Write an expression to find the cost of the jeans if the 25% discount is applied first and the $5 coupon is applied second. Does your expression represent $[f \circ g](x)$ or $[g \circ f](x)$

d) Write an expression to find the cost of the jeans if the $5 coupon is applied first and the 25% discount is applied second.

Lesson 7-4: n^{th} Roots

Objectives: To simplify radicals
To use a calculator to approximate radicals

Warm-Up: Evaluate the following

$7^2 =$ $\sqrt{49} =$

$4^3 =$ $\sqrt[3]{64} =$

$2^4 =$ $\sqrt[4]{16}$

Recall that inverse functions "undo" each other. For example, the inverse of multiplying is dividing. Finding the square root of a number and squaring a number are inverse operations. Similarly, taking the n^{th} root of a number is the inverse of raising a number to the n^{th} power.

$$\text{If } \quad a^n = b, \text{ then } a = \sqrt[n]{b}$$

We say "a is the nth root of b"

Some numbers have more than one n^{th} root. For instance, 36 has two square roots, 6 and -6. When there is more than one real root, the positive root is called the principal square root. *Whenever the index, n, is even, the radical sign indicates the positive (principal) root.*

$\sqrt{25}$ $\quad -\sqrt{25}$ $\quad \sqrt{-25}$

$\sqrt[3]{27}$ $\quad -\sqrt[3]{27}$ $\quad \sqrt[3]{-27}$

$\sqrt[4]{81}$ $\quad \sqrt[4]{-81}$ $\quad -\sqrt[4]{81}$

When you find the even n^{th} root of a <u>variable expression</u> with an even power and the answer is an odd power, you should use absolute values to make sure the answer is nonnegative.

$\sqrt{(-5)^2}$ $\quad \sqrt{x^2}$ $\quad \sqrt{(-3)^4}$ $\quad \sqrt{x^4}$

$\sqrt[3]{-8}$ $\quad \sqrt[3]{y^3}$ $\quad \sqrt[4]{(x+1)^{12}}$

In a simplified radical expression, the radicand will not contain powers that are multiples of the index.

To simplify a radical expression:

1. Factor the radicand
2. Isolate powers that are multiples of the index and remove from the radicand

Example: Simplify the following

$\sqrt{54}$

$\sqrt[3]{128}$

$\sqrt{16p^8q^7}$

$\sqrt[3]{8g^3h^8}$

$\sqrt{36r^5s^{10}}$

$\sqrt[4]{32x^7y^9}$

You can use your graphing calculator to find the nth root by selecting $\sqrt[n]{}$ from the MATH menu. Your calculator gives you an approximation of the <u>irrational number.</u>

Example: Evaluate to the nearest thousandth

$\sqrt[3]{19}$

$\sqrt[5]{891}$

Lesson 7-5: Operations with Radicals

Objective: To add, subtract, multiply radical expressions
To rationalize the denominator of radical expressions

Adding and Subtracting Radical Expressions

You can only add or subtract radical expressions with the same indices and the same radicands!!! Sometimes you can simplify the radicals to obtain like terms.

$$\sqrt{3}+5\sqrt{2}-3\sqrt{2}+6\sqrt{3}$$

$$3\sqrt{45}-5\sqrt{80}$$

$$2\sqrt{12}-3\sqrt{27}+2\sqrt{48}$$

Multiplying Radical Expressions

Radicals with the same index can be multiplied and then simplified:

$$\sqrt{12} \bullet \sqrt{3}$$

$$3\sqrt{6x^3y} \cdot 2\sqrt{24xy^2}$$

You can use FOIL to multiply "binomial radicals"

$$\left(3\sqrt{5} - 2\sqrt{3}\right)\left(2 + \sqrt{3}\right)$$

$$\left(5\sqrt{3} - 6\right)\left(5\sqrt{3} + 6\right)$$

In order for a radical expression to be considered simplified, there cannot be a radical in the denominator. The process of removing the radical from the denominator is called rationalizing.

Simplify:

$$\sqrt{\frac{1}{2}}$$

$$\sqrt{\frac{x^4}{y^5}}$$

$$\sqrt{\frac{2}{9x}}$$

$$\sqrt[3]{\frac{2}{9x}}$$

Rationalizing "binomial" radicals

Radicals like $a\sqrt{b}+c\sqrt{d}$ and $a\sqrt{b}-c\sqrt{d}$ are called conjugates. You can use conjugates to rationalize binomial denominators.

Simplify:

$$\frac{5}{4-\sqrt{5}}$$

$$\frac{1-\sqrt{3}}{5+\sqrt{3}}$$

Lesson 7-6: Fractional Exponents

Objectives: To write expressions with fractional exponents as radicals and vice-versa; To evaluate expressions with fractional exponents

Use your calculator to find: $25^{\frac{1}{2}}$ $27^{\frac{1}{3}}$ $27^{\frac{2}{3}}$

Definition of Fractional Exponents:

$$b^{\frac{1}{n}} = \sqrt[n]{b}$$

$$b^{\frac{m}{n}} = \sqrt[n]{b^m}$$

Example: Write each expression in radical form.

$a^{\frac{1}{4}}$ $x^{\frac{1}{2}}$ $y^{\frac{2}{3}}$

Example: Write each expression using fractional exponents

$\sqrt[3]{y}$ $\sqrt{6}$ $\sqrt[4]{x^3}$

When evaluating expressions with fractional exponents, it's usually easiest to do the root first, and then the power.

Evaluate:

$32^{\frac{2}{5}}$ $(-64)^{\frac{2}{3}}$ $(-9)^{\frac{3}{2}}$

Don't forget the negative exponent rule!

Evaluate:

$49^{\frac{-1}{2}}$ $\left(\frac{1}{243}\right)^{\frac{-3}{5}}$ $256^{\frac{-3}{4}}$

All of the Laws of Exponents still apply to fractional exponents.

An expression with fractional exponents is simplified when

- It has no negative exponents
- It has no fractional exponents in the denominator
- The index of any remaining radical is as small as possible

Simplify:

$y^{\frac{1}{7}} \cdot y^{\frac{4}{7}}$

$x^{\frac{3}{10}} \cdot x^{\frac{7}{10}}$

$\dfrac{x^{\frac{3}{4}}}{x^{\frac{1}{8}}}$

$c^{\frac{2}{3}}c^{\frac{3}{4}}c^{\frac{1}{2}}$

In order to fully simplify a radical expression, it is sometimes easier to change it to a fractional exponent first. If the original problem is a radical expression, you should change it back to that form for your final answer.

Simplify by rewriting as a fractional exponent:

$\sqrt[4]{9}$

$\sqrt[6]{36x^4y^2}$

$\sqrt{13} \bullet \sqrt[3]{13^2}$

$\sqrt{\sqrt[3]{49}}$

Lesson 7-7: Radical Equations and Inequalities

Objective: To solve equations and inequalities containing radicals

Equations that have variables under the radicand are called radical equations. When you solve a radical equation, it is important that you check your solution. Sometimes you will get a number that does not satisfy the original equation. This is called an **extraneous solution.**

To solve a radical equation with one radical:

1. Isolate the radical
2. Eliminate the radical by squaring (or cubing, etc.) each side of the equation.
3. Solve the resulting equation.
4. Check for extraneous solutions

Solve for x.

$$4\sqrt{2x+1} = 24$$

$$\sqrt{x+1}+4=2$$

$$(3x+1)^{\frac{1}{3}}+5=0$$

If the equation has two radicals, it will be easier to square both sides first, and then isolate the remaining radical. Be sure to check for extraneous solutions!

$$\sqrt{x-15}=3-\sqrt{x}$$

$$1+\sqrt{x+5}=\sqrt{x+12}$$

Radical Inequalities

Find the values of x that solve the inequality $2+\sqrt{4x-4}\le 6$ and show your solutions on a number line.

First, we know the radicand must be nonnegative. (Why?)

Solve the inequality $4x - 4 \ge 0$

Now solve $2+\sqrt{4x-4}\le 6$

Lesson 7-7, continued

Objective: To use the graphing calculator to solve radical equations and inequalities

Solve $2\sqrt{x} = \sqrt{x+2} + 1$

1. Graph $y_1 = 2\sqrt{x}$ and $y_2 = \sqrt{x+2} + 1$
2. Use the 2nd CALC intersect to find the intersection and report the value of x.

Solve $\sqrt{3x-6} < 3$

1. Graph $y_1 = \sqrt{3x-6}$ and $y_2 = 3$
2. Use the 2nd CALC intersect to find the intersection of y_1 and y_2
3. Determine the values of x which are greater than the starting point of the graph and less than the POI

Word Problem Review

1. The volume of a sphere is given by the equation $V = \frac{4}{3}\pi r^3$.

   a) Solve this formula for r.

   b) Rewrite the formula for r using fractional exponents.

   c) A men's basketball holds 455 cubic inches of air. What is its radius?

2. The distance d traveled by a falling object in t seconds is given by $192t^2 = d$

   a) Solve the formula for t.

   b) A student is holding a ruler at the top and lets it falls. The ruler fell 6 inches before she caught it. How long did it take for her to catch the ruler? (This is called the reaction time).

3. Your friend says that $\sqrt{a^2 + b^2} = a + b$. Do you agree? Why or why not?